ME 141

Engineering Mechanics

Lecture 10: Kinetics of particles: Newton's 2nd Law

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Courtesy: Vector Mechanics for Engineers, Beer and Johnston

Introduction

$$\Sigma \mathbf{F} = m\mathbf{a}$$

- **Newton's Second Law of Motion**
  - If the *resultant force* acting on a particle is not zero, the particle will have an acceleration *proportional to the magnitude of resultant* and in the *direction of the resultant*.
  - Must be expressed with respect to a *Newtonian (or inertial) frame of reference*, i.e., one that is not accelerating or rotating.
  - This form of the equation is for a constant mass system

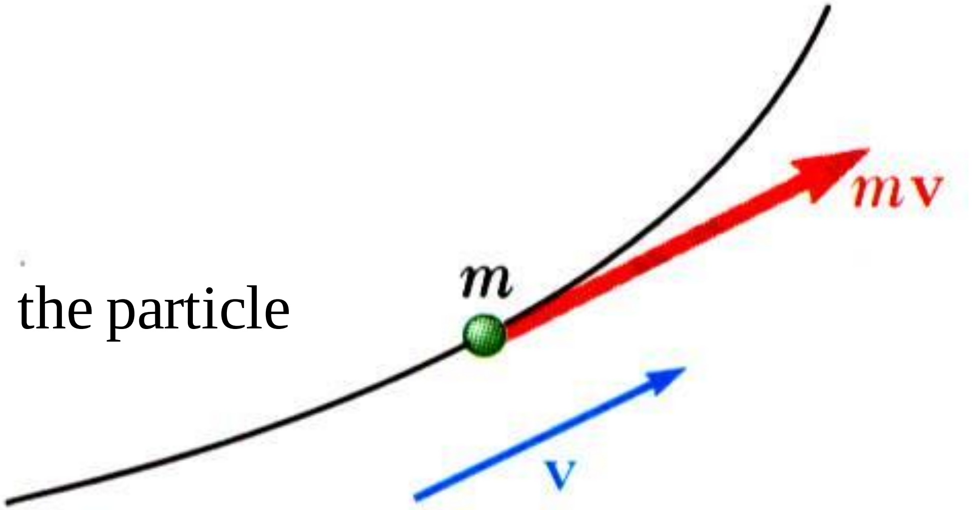
Linear Momentum of a Particle

- Replacing the acceleration by the derivative of the velocity yields

$$\sum \vec{F} = m\frac{d\vec{v}}{dt}$$

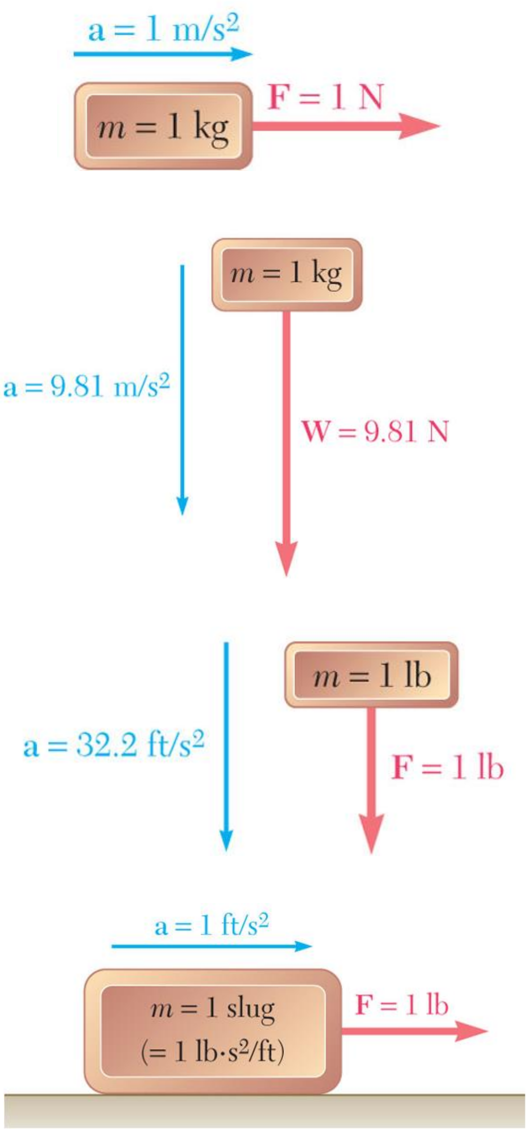
$$= \frac{d}{dt}(m\vec{v}) = \frac{d\vec{L}}{dt}$$

$$\vec{L} = \text{linear momentum of the particle}$$



- *Linear Momentum Conservation Principle*:
If the resultant force on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction.

Systems of Units



- Of the units for the four primary dimensions (force, mass, length, and time), three may be chosen arbitrarily. The fourth must be compatible with Newton's 2nd Law.

- *International System of Units* (SI Units): base units are the units of length (m), mass (kg), and time (second). The unit of force is derived,

$$1\,\mathrm{N} = (1\,\mathrm{kg})\left(1\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 1\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$$

- *U.S. Customary Units*: base units are the units of force (lb), length (m), and time (second). The unit of mass is derived,

$$1\,\mathrm{lbm} = \frac{1\,\mathrm{lb}}{32.2\,\mathrm{ft}/\mathrm{s}^2} \qquad 1\,\mathrm{slug} = \frac{1\,\mathrm{lb}}{1\,\mathrm{ft}/\mathrm{s}^2} = 1\frac{\mathrm{lb}\cdot\mathrm{s}^2}{\mathrm{ft}}$$

Equations of Motion

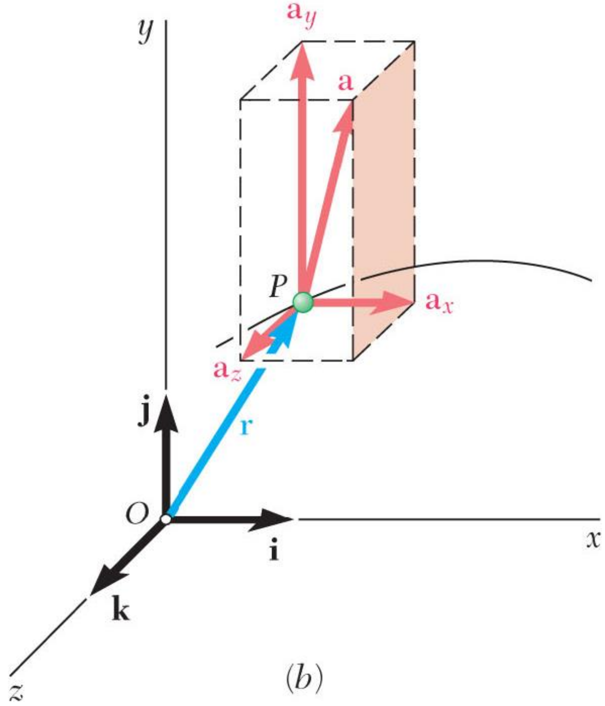
- Newton's second law $$\sum \vec{F} = m\vec{a}$$

- Can use scalar component equations, e.g., for rectangular components,

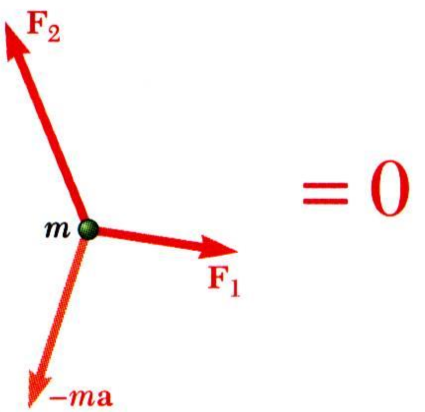
$$\sum\left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right) = m\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$

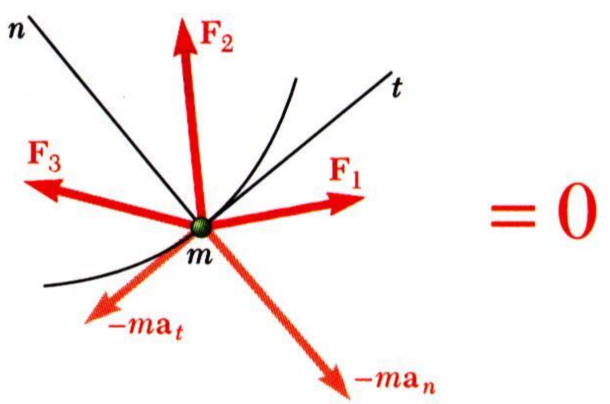
$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

$$\sum F_x = m\ddot{x} \qquad \sum F_y = m\ddot{y} \qquad \sum F_z = m\ddot{z}$$



Dynamic Equilibrium





- Alternate expression of Newton's second law,

$$\sum \vec{F} - m\vec{a} = 0$$

$$-m\vec{a} \equiv \textit{inertial vector}$$

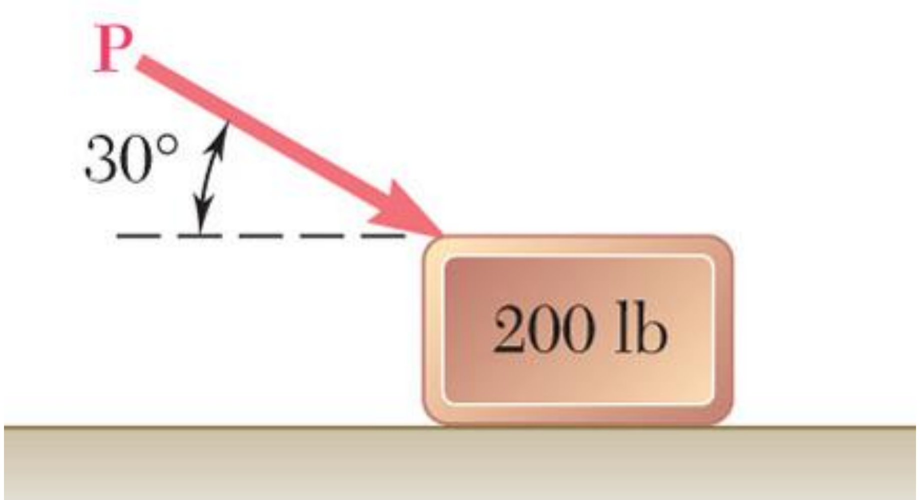
- With the inclusion of the inertial vector, the system of forces acting on the particle is equivalent to zero. The particle is in *dynamic equilibrium*.

- Methods developed for particles in static equilibrium may be applied, e.g., coplanar forces may be represented with a closed vector polygon.

- Inertia vectors are often called *inertial forces* as they measure the resistance that particles offer to changes in motion, i.e., changes in speed or direction.

- Inertial forces may be conceptually useful but are not like the contact and gravitational forces found in statics.

Sample Problem 12.1



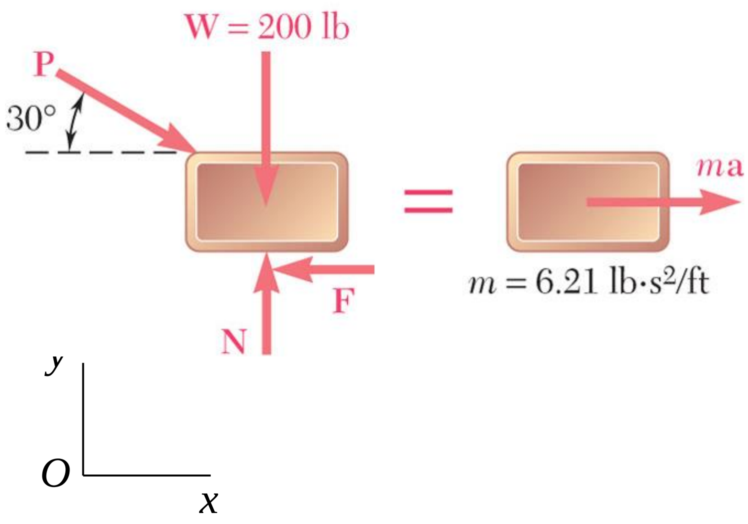
A 200-lb block rests on a horizontal plane. Find the magnitude of the force ***P*** required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and plane is $\mu_k = 0.25$.

SOLUTION:

- Resolve the equation of motion for the block into two rectangular component equations.

- Unknowns consist of the applied force ***P*** and the normal reaction ***N*** from the plane. The two equations may be solved for these unknowns.

Sample Problem 12.1



$$m = \frac{W}{g} = \frac{200\,\text{lb}}{32.2\,\text{ft}/\text{s}^2}$$

$$= 6.21 \frac{\text{lb}\cdot\text{s}^2}{\text{ft}}$$

$$F = \mu_k N$$

$$= 0.25N$$

SOLUTION:

- Resolve the equation of motion for the block into two rectangular component equations.

$$\sum F_x = ma:$$

$$P\cos 30° - 0.25N = \left(6.21\,\text{lb}\cdot\text{s}^2/\text{ft}\right)\left(10\,\text{ft}/\text{s}^2\right)$$

$$= 62.1\,\text{lb}$$

$$\sum F_y = 0:$$

$$N - P\sin 30° - 200\,\text{lb} = 0$$

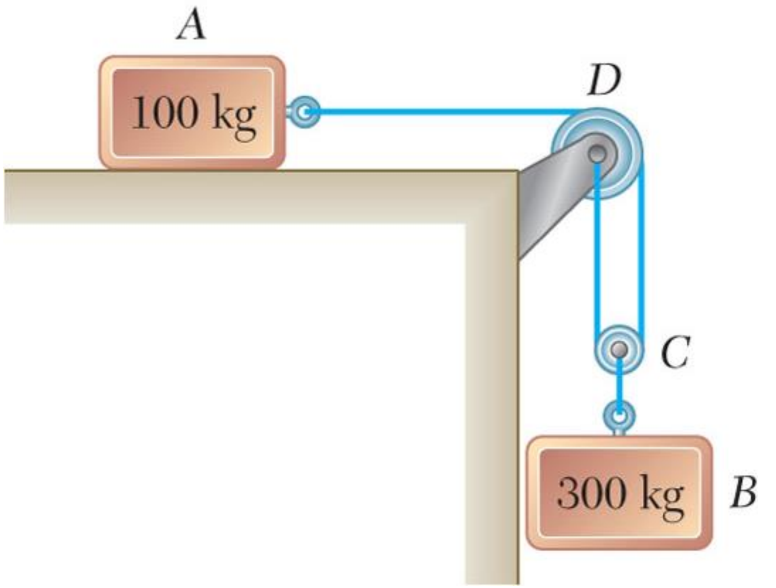
- Unknowns consist of the applied force ***P*** and the normal reaction ***N*** from the plane. The two equations may be solved for these unknowns.

$$N = P\sin 30° + 200\,\text{lb}$$

$$P\cos 30° - 0.25\left(P\sin 30° + 200\,\text{lb}\right) = 62.1\,\text{lb}$$

$$\boxed{P = 151\,\text{lb}}$$

Sample Problem 12.3

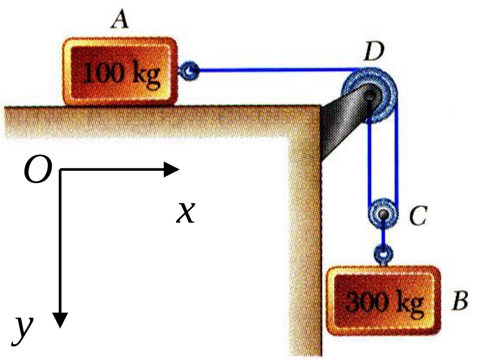


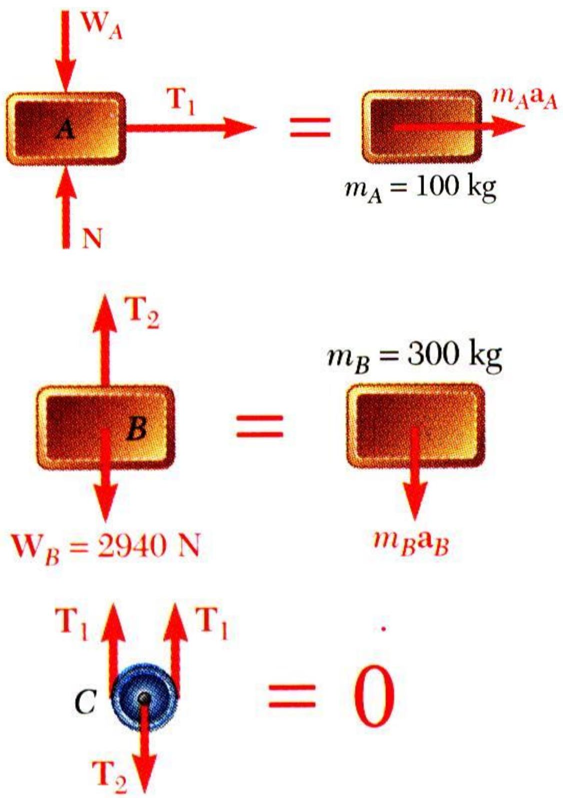
The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.

SOLUTION:

- Write the kinematic relationships for the dependent motions and accelerations of the blocks.
- Write the equations of motion for the blocks and pulley.
- Combine the kinematic relationships with the equations of motion to solve for the accelerations and cord tension.

Sample Problem 12.3





SOLUTION:

- Write the kinematic relationships for the dependent motions and accelerations of the blocks.

$$y_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

- Write equations of motion for blocks and pulley.

$$\sum F_x = m_A a_A :$$

$$T_1 = (100\,\text{kg}) a_A$$

$$\sum F_y = m_B a_B :$$

$$m_B g - T_2 = m_B a_B$$

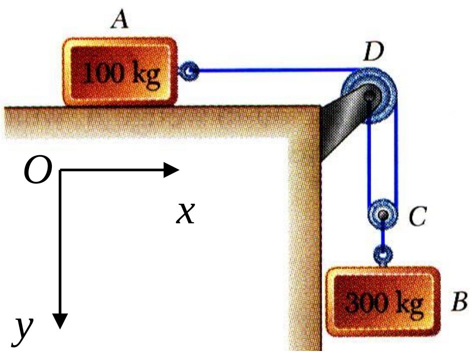
$$(300\,\text{kg})\left(9.81\,\text{m/s}^2\right) - T_2 = (300\,\text{kg}) a_B$$

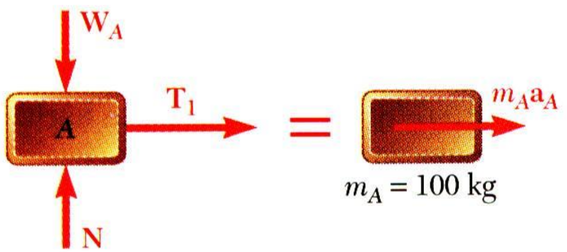
$$T_2 = 2940\,\text{N} - (300\,\text{kg}) a_B$$

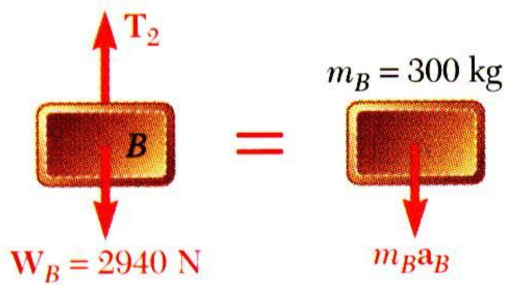
$$\sum F_y = m_C a_C = 0 :$$

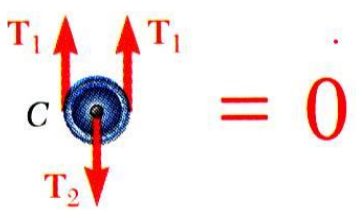
$$T_2 - 2T_1 = 0$$

Sample Problem 12.3









- Combine kinematic relationships with equations of motion to solve for accelerations and cord tension.

$$y_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

$$T_1 = (100\,\text{kg})a_A$$

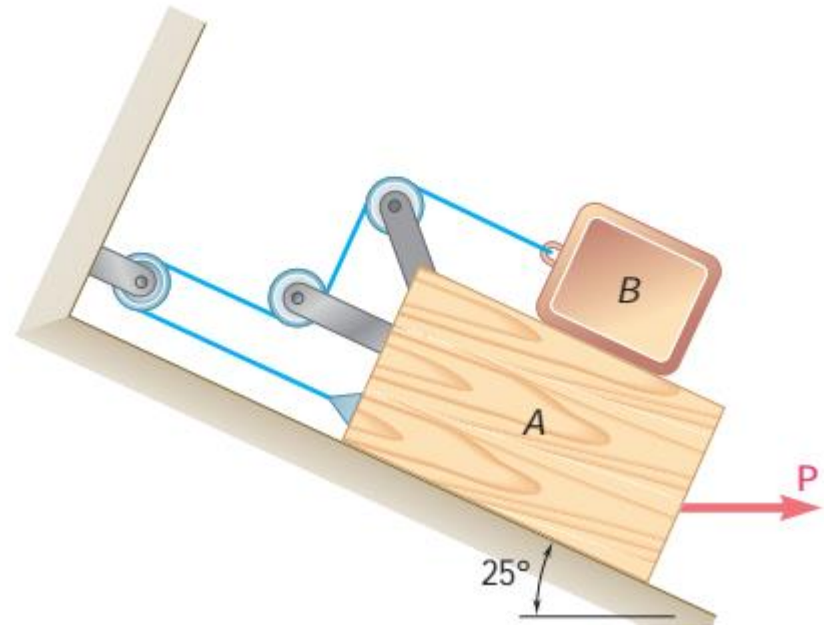
$$T_2 = 2940\,\text{N} - (300\,\text{kg})a_B$$
$$= 2940\,\text{N} - (300\,\text{kg})\left(\tfrac{1}{2} a_A\right)$$

$$T_2 - 2T_1 = 0$$
$$2940\,\text{N} - (150\,\text{kg})a_A - 2(100\,\text{kg})a_A = 0$$

$$\boxed{\begin{aligned} a_A &= 8.40\,\text{m/s}^2 \\ a_B &= \tfrac{1}{2} a_A = 4.20\,\text{m/s}^2 \\ T_1 &= (100\,\text{kg})a_A = 840\,\text{N} \\ T_2 &= 2T_1 = 1680\,\text{N} \end{aligned}}$$

Prob # 12.19

Block *A* has a mass of 40 kg, and block *B* has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 40$ N , determine (*a*) the acceleration
of block *B,* (*b*) the tension in the cord

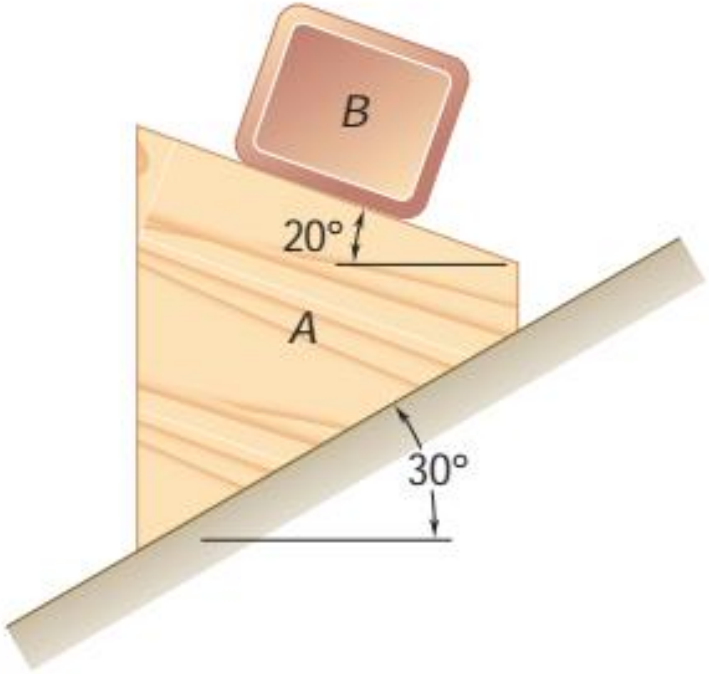


Prob # 12.35

Block *B* of mass 10 kg rests as shown on the upper surface of a 22-kg wedge *A*. Knowing that the system is released from rest and neglecting friction, determine

(*a*) the acceleration of *B*,

(*b*) The velocity of *B* relative to *A* at $t = 0.5$ s.



Kinetics: Normal and Tangential Coordinates

Aircraft and roller coasters can both experience large normal forces during turns.

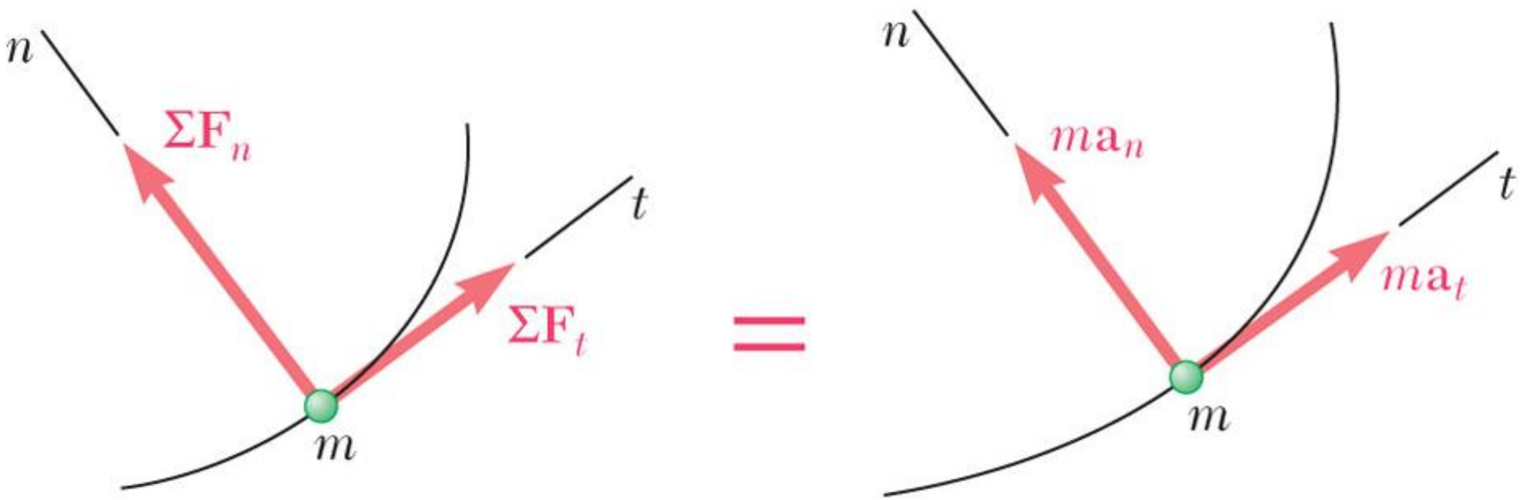
Equations of Motion

- Newton's second law $$\sum \vec{F} = m\vec{a}$$

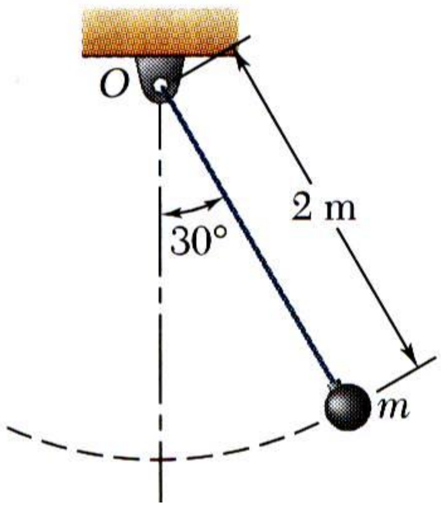
- For tangential and normal components,

$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$

$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$



Sample Problem 12.5

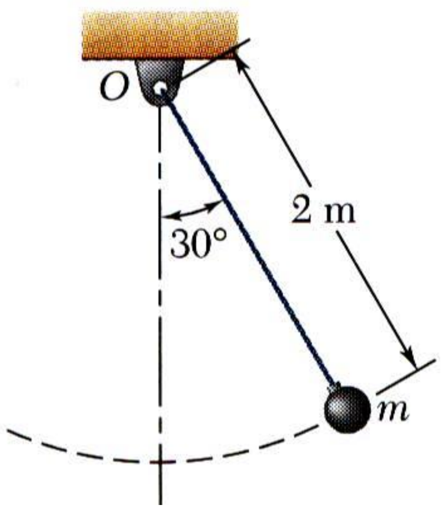


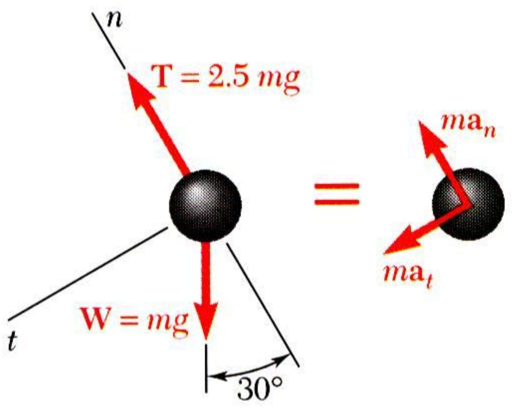
The bob of a 2-m pendulum describes an arc of a circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and acceleration of the bob in that position.

SOLUTION:

- Resolve the equation of motion for the bob into tangential and normal components.
- Solve the component equations for the normal and tangential accelerations.
- Solve for the velocity in terms of the normal acceleration.

Sample Problem 12.5





SOLUTION:

- Resolve the equation of motion for the bob into tangential and normal components.
- Solve the component equations for the normal and tangential accelerations.

$$\sum F_t = ma_t : \qquad mg\sin 30° = ma_t$$

$$a_t = g\sin 30°$$

$$\boxed{a_t = 4.9\,\mathrm{m/s^2}}$$

$$\sum F_n = ma_n : \qquad 2.5mg - mg\cos 30° = ma_n$$

$$a_n = g(2.5 - \cos 30°)$$

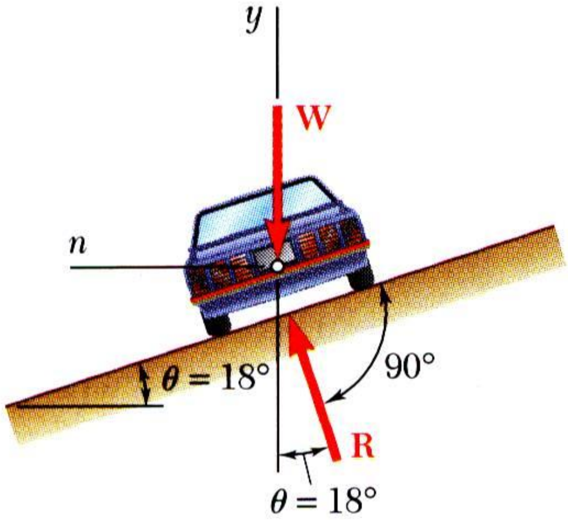
$$\boxed{a_n = 16.03\,\mathrm{m/s^2}}$$

- Solve for velocity in terms of normal acceleration.

$$a_n = \frac{v^2}{\rho} \qquad v = \sqrt{\rho a_n} = \sqrt{(2\,\mathrm{m})(16.03\,\mathrm{m/s^2})}$$

$$\boxed{v = \pm 5.66\,\mathrm{m/s}}$$

Sample Problem 12.6



Determine the rated speed of a highway curve of radius $\rho = 400$ ft banked through an angle $\theta = 18^{o}$. The rated speed of a banked highway curve is the speed at which a car should travel if no lateral friction force is to be exerted at its wheels.

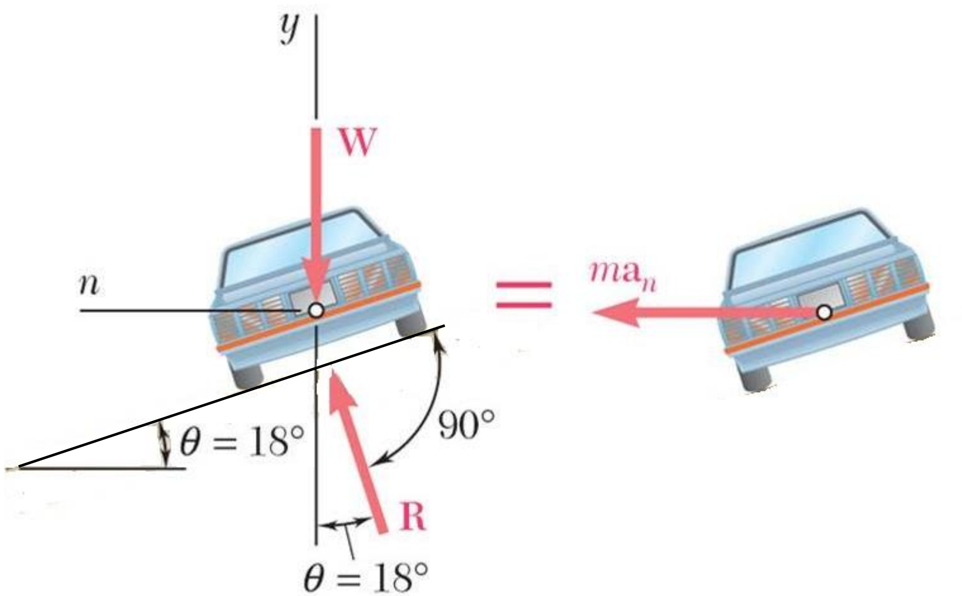
SOLUTION:

- The car travels in a horizontal circular path with a normal component of acceleration directed toward the center of the path.The forces acting on the car are its weight and a normal reaction from the road surface.

- Resolve the equation of motion for the car into vertical and normal components.

- Solve for the vehicle speed.

Sample Problem 12.6



- The car travels in a horizontal circular path with a normal component of acceleration directed toward the center of the path.The forces acting on the car are its weight and a normal reaction from the road surface.

- Resolve the equation of motion for the car into vertical and normal components.

$$\sum F_y = 0: \qquad R\cos\theta - W = 0$$

$$R = \frac{W}{\cos\theta}$$

$$\sum F_n = ma_n: \qquad R\sin\theta = \frac{W}{g}a_n$$

$$\frac{W}{\cos\theta}\sin\theta = \frac{W}{g}\frac{v^2}{\rho}$$

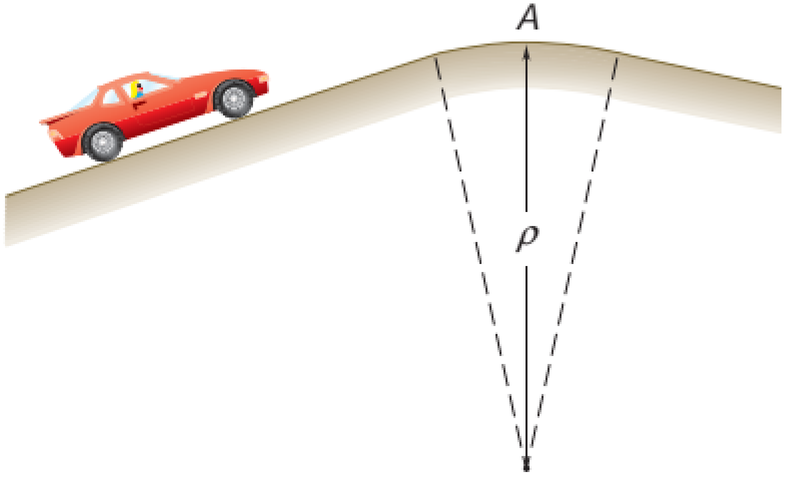
- Solve for the vehicle speed.

$$v^2 = g\rho\tan\theta$$
$$= \left(32.2\,\mathrm{ft/s^2}\right)\left(400\,\mathrm{ft}\right)\tan 18°$$

$$\boxed{v = 64.7\,\mathrm{ft/s} = 44.1\,\mathrm{mi/h}}$$

Prob # 12.45

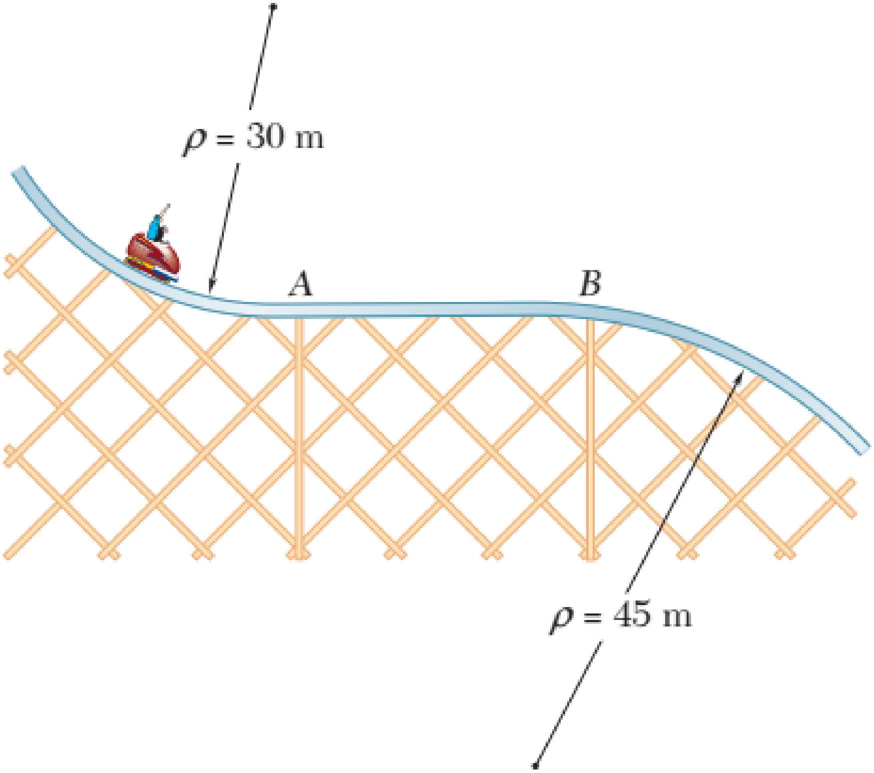
During a high-speed chase, a 2400-lb sports car traveling at a speed of 100 mi/h just loses contact with the road as it reaches the crest *A* of a hill. (*a*) Determine the radius of curvature r of the vertical profile of the road at *A*. (*b*) Using the value of r found in part *a*, determine the force exerted on a 160-lb driver by the seat of his 3100-lb car as the car, traveling at a constant speed of 50 mi/h, passes through *A*.



Prob # 12.47

The roller-coaster track shown is contained in a vertical plane. The portion of track between *A* and *B* is straight and horizontal, while the portions to the left of *A* and to the right of *B* have radii of curvature as indicated. A car is traveling at a speed of 72 km/h when the brakes are suddenly applied, causing the wheels of the car to slide on the track (μ_k= 0.20).

Determine the initial deceleration of the car if the brakes are applied as the car
(a) has almost reached *A*,
(*b*) is traveling between *A* and *B*,
(*c*) has just passed *B*.



Prob # 12.49

A series of small packages, each with a mass of 0.5 kg, are discharged from a conveyor belt as shown. Knowing that the coefficient of static friction between each package and the conveyor belt is 0.4, determine

(a) the force exerted by the belt on the package just after it has passed point *A*,

(b) the angle θ defining the point *B* where the packages first *slip* relative to the belt.